Math Virtual Learning

Geometry/Honors Geometry

May 21, 2020

Geometry

Lesson: May 21, 2020

Objective/Learning Target:

Calculate conditional probability of events.

Bell Ringer: Suppose that we are going to roll two fair 6-sided dice. Find the probability that both dice show an even number.

Bell Ringer Answer: P (both even) = 1/4

Let’s Get Started: Go through the following slides and try the example problems.

Conditional Probability

The ***conditional probability*** of an event B is the probability that the event will occur given the knowledge that an event A has already occurred. This probability is written $P(B|A)$, notation for the *probability of B given A*. In the case where events A and B are *independent* (where event A has no effect on the probability of event B), the conditional probability of event B given event A is simply the probability of event B, that is $P(B)$.

If events A and B are not independent, then the probability of the *intersection of A and B* (the probability that both events occur) is defined by
$P(A \text{ and } B) = P(A)P(B|A)$.

From this definition, the conditional probability $P(B|A)$ is easily obtained by dividing by $P(A)$:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Note: This expression is only valid when P(A) is greater than 0.

Example 1.15

I roll a fair die. Let A be the event that the outcome is an odd number, i.e., $A = \{1,3,5\}$. Also let B be the event that the outcome is less than or equal to 3, i.e., $B = \{1,2,3\}$. What is the probability of A, $P(A)$? What is the probability of A given B, $P(A|B)$?

This is a finite sample space, so

$$P(A) = \frac{|A|}{|S|} = \frac{|\{1,3,5\}|}{6} = \frac{1}{2}.$$

Now, let's find the conditional probability of A given that B occurred. If we know B has occurred, the outcome must be among $\{1,2,3\}$. For A to also happen the outcome must be in $A \cap B = \{1,3\}$. Since all die rolls are equally likely, we argue that $P(A|B)$ must be equal to

$$P(A|B) = \frac{|A \cap B|}{|B|} = \frac{2}{3}.$$

Example: Drawing 2 Kings from a Deck

Event A is drawing a King first, and **Event B** is drawing a King second.

Answer Key:

Here you will find the answers to the previous four questions.
Check your answers below.

For the first card the chance of drawing a King is 4 out of 52 (there are 4 Kings in a deck of 52 cards):

$$P(A) = 4/52$$

But after removing a King from the deck the probability of the 2nd card drawn is **less** likely to be a King (only 3 of the 51 cards left are Kings):

$$P(B|A) = 3/51$$

And so:

$$\mathbf{P(A\ and\ B)} = \mathbf{P(A) \times P(B|A)} = (4/52) \times (3/51) = 12/2652 = \mathbf{1/221}$$

So the chance of getting 2 Kings is 1 in 221, or about 0.5%

Additional Resources:

Click on the link below to get additional practice and to check your understanding!

Conditional Probability Practice